Tracking

- Establish where an object is, other aspects of state, using time sequence
  - Biggest problem -- Data Association
- Key ideas
  - Tracking by detection
  - Tracking through flow

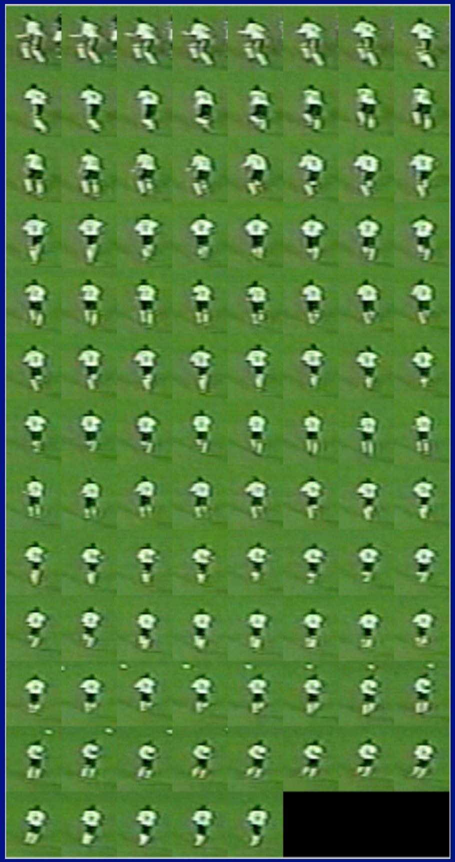
Track by detection (simple form)

- Assume
  - a fairly reliable detector (e.g. faces; back of heads)
  - detections that are well spaced in images (or have distinctive properties)
    - e.g. news anchors; heads in public
- Track by
  - detect in each frame
  - link using matching algorithm
    - measurements with no track? create new track
    - tracks with no measurement? wait, then reap

Track by flow (simple form)

- Assume
  - appearance unknown (but domain, parametric flow model known)
  - optic flow assumptions, as before
- Initialize
  - mark out domain
- Track
  - choose flow model parameters that align domain in pic n with n+1 best
  - push domain through flow model

Efros et al, 03



Efros et al, 03

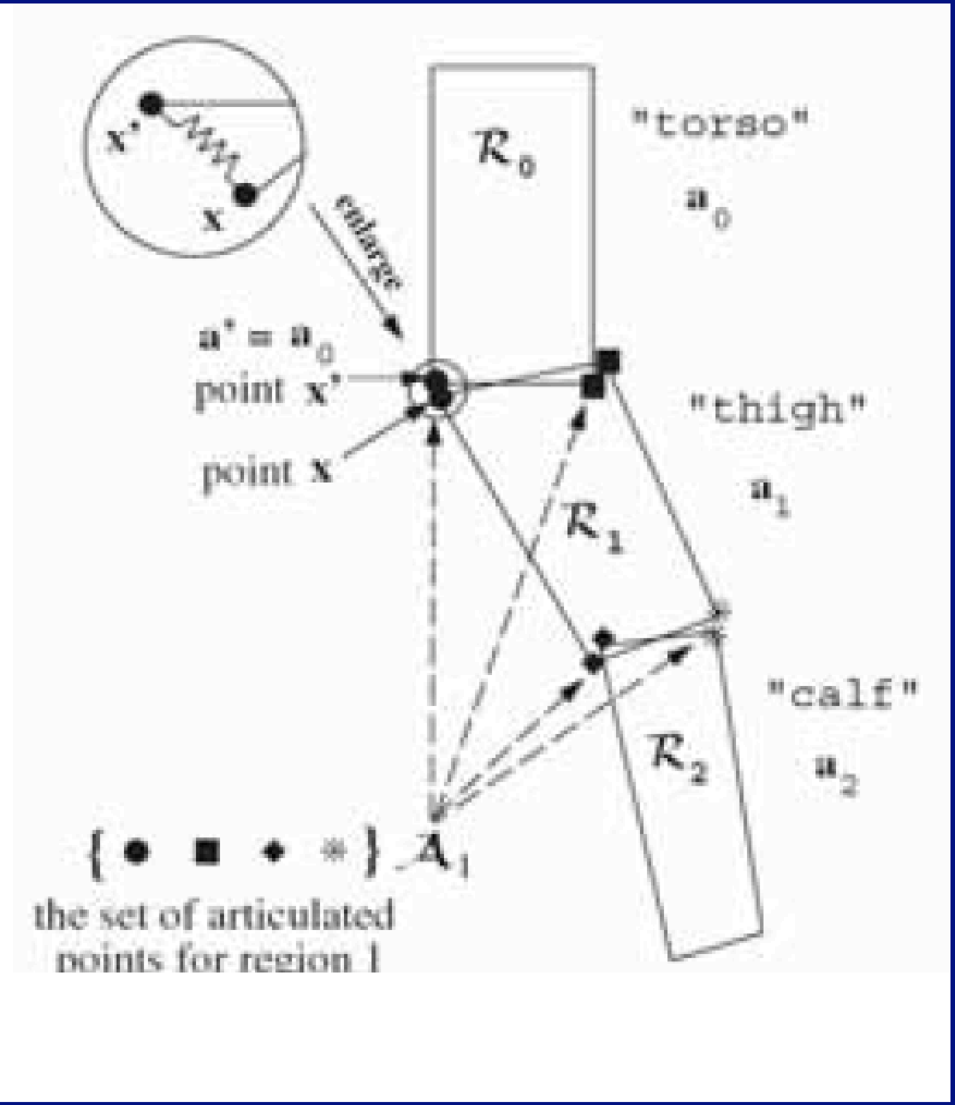


Figure from Ju, Black and Yacoob, “Cardboard people”

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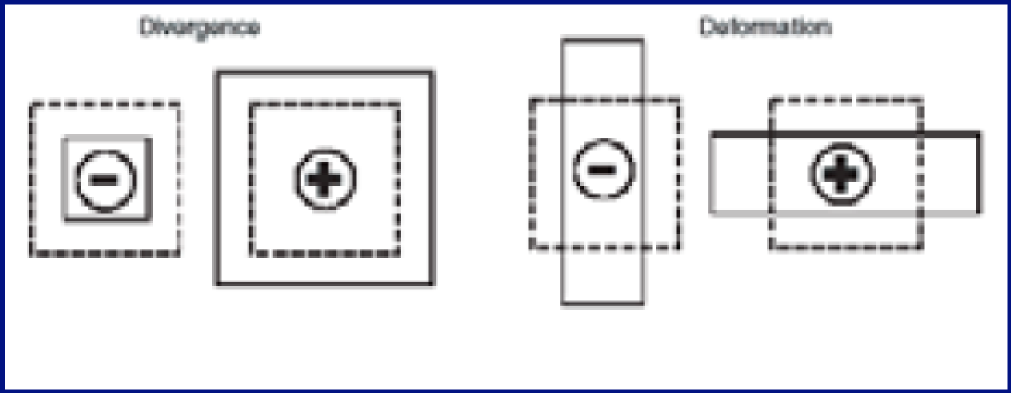


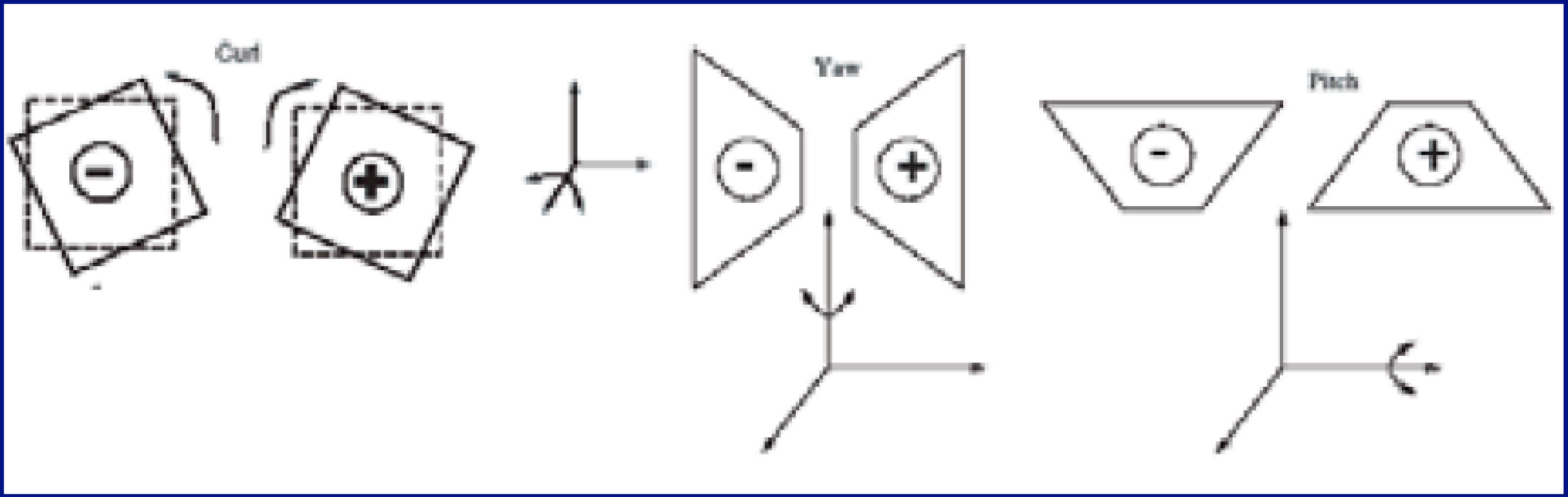
$\mathbf{a} = (0, 1, 0, 0, 0, 1, 0, 0)$

$\mathbf{a} = (0, 1, 0, 0, 0, -1, 0, 0).$

$$(u(\mathbf{x}), v(\mathbf{x}))^T =$$

$$(a_0 + a_1 x + a_2 y + a_6 x^2 + a_7 xy, a_3 + a_4 x + a_5 y + a_6 xy + a_7 y^2)$$

Figure from Ju, Black and Yacoob, “Cardboard people”



$(0, 0, -1, 0, 1, 0, 0, 0)$ $(0, 0, 0, 0, 0, 1, 0)$ $(0, 0, 0, 0, 0, 0, 1)$

Figure from Ju, Black and Yacoob, “Cardboard people”

Dangers

- Loss of track
  - small errors accumulate in model of appearance
  - DRIFT
- Appearance often isn't constant

Tracking - more formal view

- Very general model:
  - We assume there are moving objects, which have an underlying state X
  - There are observations Y, some of which are functions of this state
  - There is a clock
    - at each tick, the state changes
    - at each tick, we get a new observation
- Examples
  - object is ball, state is 3D position+velocity, observations are stereo pairs
  - object is person, state is body configuration, observations are frames, clock is in camera (30 fps)

Tracking - Probabilistic formulation

- Given
  - P(X_i-1|Y_0, ..., Y_i-1)
    - "Prior"
- We should like to know
  - P(X_i|Y_0, ..., Y_i-1)
    - "Predictive distribution"
  - P(X_i|Y_0, ..., Y_i)
    - "Posterior"

The three main issues in tracking

- **Prediction:** we have seen $\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1}$ — what state does this set of measurements predict for the i'th frame? to solve this problem, we need to obtain a representation of $P(\boldsymbol{X}_i | \boldsymbol{Y}_0 = \boldsymbol{y}_0, \ldots, \boldsymbol{Y}_{i-1} = \boldsymbol{y}_{i-1})$.
- **Data association:** Some of the measurements obtained from the i-th frame may tell us about the object's state. Typically, we use $P(\boldsymbol{X}_i | \boldsymbol{Y}_0 = \boldsymbol{y}_0, \ldots, \boldsymbol{Y}_{i-1} = \boldsymbol{y}_{i-1})$ to identify these measurements.
- **Correction:** now that we have $\boldsymbol{y}_i$ — the relevant measurements — we need to compute a representation of $P(\boldsymbol{X}_i | \boldsymbol{Y}_0 = \boldsymbol{y}_0, \ldots, \boldsymbol{Y}_i = \boldsymbol{y}_i)$.

Key assumptions:

- **Only the immediate past matters:** formally, we require

$$P(\boldsymbol{X}_i|\boldsymbol{X}_1, \ldots, \boldsymbol{X}_{i-1}) = P(\boldsymbol{X}_i|\boldsymbol{X}_{i-1})$$

  This assumption hugely simplifies the design of algorithms, as we shall see; furthermore, it isn't terribly restrictive if we're clever about interpreting $\boldsymbol{X}_i$ as we shall show in the next section.

- **Measurements depend only on the current state:** we assume that $\boldsymbol{Y}_i$ is conditionally independent of all other measurements given $\boldsymbol{X}_i$. This means that

$$P(\boldsymbol{Y}_i, \boldsymbol{Y}_j, \ldots \boldsymbol{Y}_k|\boldsymbol{X}_i) = P(\boldsymbol{Y}_i|\boldsymbol{X}_i)P(\boldsymbol{Y}_j, \ldots, \boldsymbol{Y}_k|\boldsymbol{X}_i)$$

  Again, this isn't a particularly restrictive or controversial assumption, but it yields important simplifications.

Tracking as Induction - base case

Firstly, we assume that we have $P(\boldsymbol{X}_0)$

$$
\begin{aligned}
P(\boldsymbol{X}_0|\boldsymbol{Y}_0=\boldsymbol{y}_0) &= \frac{P(\boldsymbol{y}_0|\boldsymbol{X}_0)P(\boldsymbol{X}_0)}{P(\boldsymbol{y}_0)} \\
&= \frac{P(\boldsymbol{y}_0|\boldsymbol{X}_0)P(\boldsymbol{X}_0)}{\int P(\boldsymbol{y}_0|\boldsymbol{X}_0)P(\boldsymbol{X}_0)d\boldsymbol{X}_0} \\
&\propto P(\boldsymbol{y}_0|\boldsymbol{X}_0)P(\boldsymbol{X}_0)
\end{aligned}
$$

Tracking as induction - induction step

Given

$P(\boldsymbol{X}_{i-1}|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})$.

Prediction

Prediction involves representing

$$P(\boldsymbol{X}_i|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})$$

Our independence assumptions make it possible to write

$$\begin{aligned}
P(\boldsymbol{X}_i|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1}) &= \int P(\boldsymbol{X}_i, \boldsymbol{X}_{i-1}|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1}) d\boldsymbol{X}_{i-1} \\
&= \int P(\boldsymbol{X}_i|\boldsymbol{X}_{i-1}, \boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1}) P(\boldsymbol{X}_{i-1}|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1}) d\boldsymbol{X}_{i-1} \\
&= \int P(\boldsymbol{X}_i|\boldsymbol{X}_{i-1}) P(\boldsymbol{X}_{i-1}|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1}) d\boldsymbol{X}_{i-1}
\end{aligned}$$

Tracking as induction - induction step

Correction

Correction involves obtaining a representation of

$$P(\boldsymbol{X}_i|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_i)$$

Our independence assumptions make it possible to write

$$\begin{aligned}
P(\boldsymbol{X}_i|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_i) &= \frac{P(\boldsymbol{X}_i, \boldsymbol{y}_0, \ldots, \boldsymbol{y}_i)}{P(\boldsymbol{y}_0, \ldots, \boldsymbol{y}_i)} \\
&= \frac{P(\boldsymbol{y}_i|\boldsymbol{X}_i, \boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})P(\boldsymbol{X}_i|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})P(\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})}{P(\boldsymbol{y}_0, \ldots, \boldsymbol{y}_i)} \\
&= P(\boldsymbol{y}_i|\boldsymbol{X}_i)P(\boldsymbol{X}_i|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})\frac{P(\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})}{P(\boldsymbol{y}_0, \ldots, \boldsymbol{y}_i)} \\
&= \frac{P(\boldsymbol{y}_i|\boldsymbol{X}_i)P(\boldsymbol{X}_i|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})}{\int P(\boldsymbol{y}_i|\boldsymbol{X}_i)P(\boldsymbol{X}_i|\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{i-1})d\boldsymbol{X}_i}
\end{aligned}$$

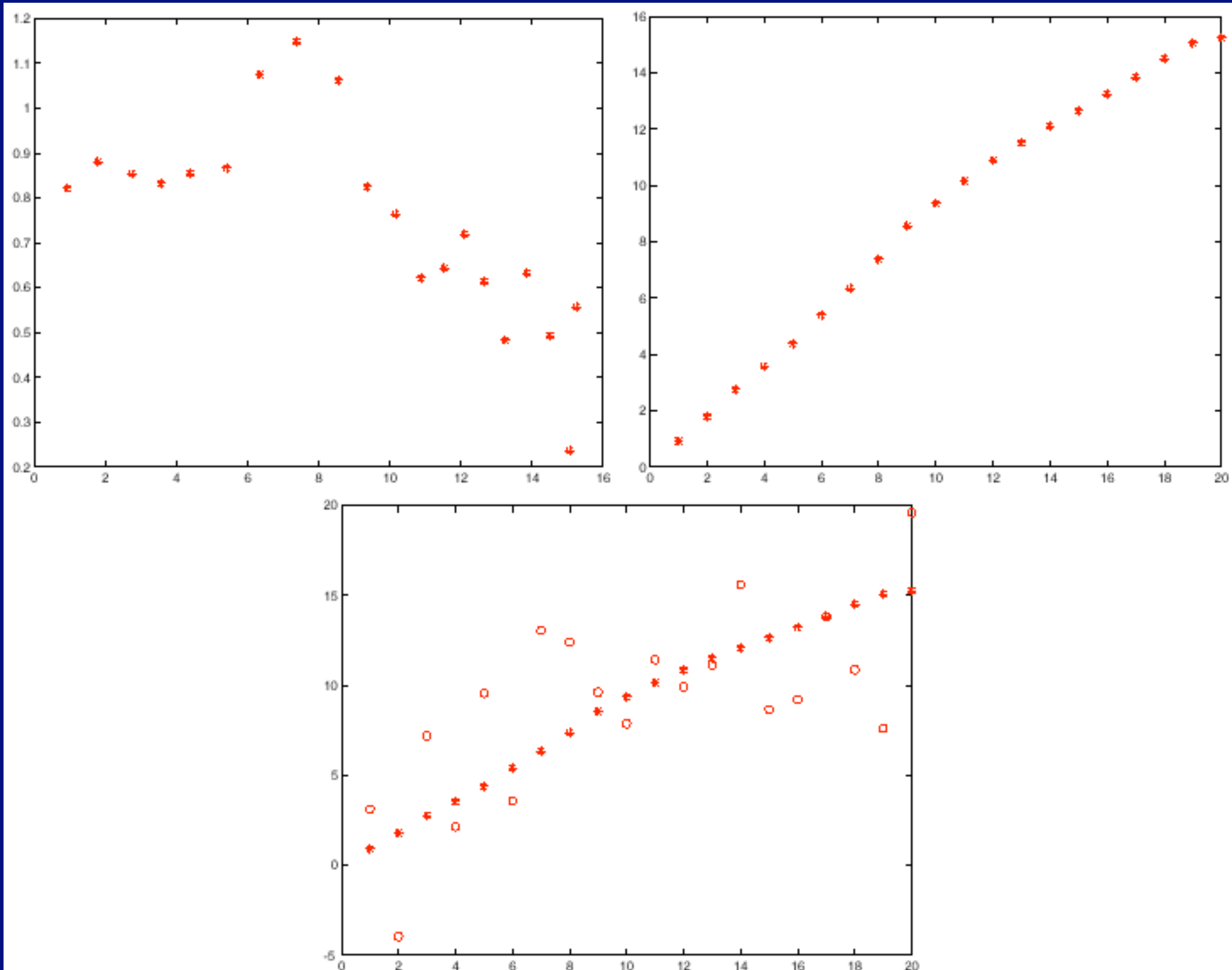
Linear Dynamic Models

$$x_i \sim N(\mathcal{D}_i \boldsymbol{x}_{i-1}; \Sigma_{d_i})$$

$$y_i \sim N(\mathcal{M}_i \boldsymbol{x}_i; \Sigma_{m_i})$$

Examples

- Drifting points
  - Observability
- Points moving with constant velocity
- Points moving with constant acceleration
- Periodic motion
- Etc.


1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0 2 4 6 8 10 12 14 16
16
14
12
10
8
6
4
2
0
0 2 4 6 8 10 12 14 16 18 20
20
15
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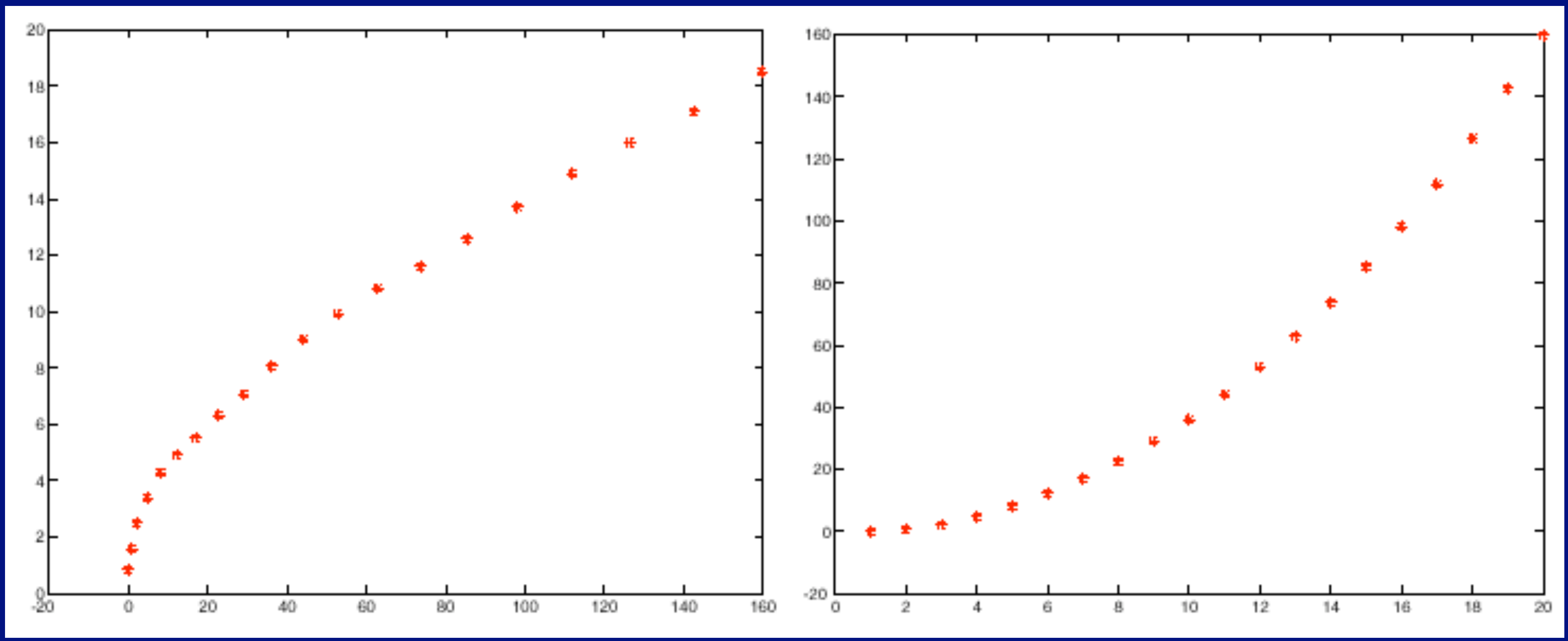
-5
0 2 4 6 8 10 12 14 16 18 20



20
18
16
14
12
10
8
6
4
2
0
-20
0
20
40
60
80
100
120
140
160
160
140
120
100
80
60
40
20
0
-20
0
2
4
6
8
10
12
14
16
18
20

The Kalman Filter

- Key ideas:
  - Linear models interact uniquely well with Gaussian noise - make the prior Gaussian, everything else Gaussian and the calculations are easy
  - Gaussians are really easy to represent --- once you know the mean and covariance, you're done

The Kalman Filter in 1D

- Dynamic Model

$$x_i \sim N(d_i x_{i-1}, \sigma_{d_i}^2)$$

$$y_i \sim N(m_i x_i, \sigma_{m_i}^2)$$

- Notation

mean of $P(X_i|y_0, \ldots, y_{i-1})$ as $\overline{X}_i^-$

mean of $P(X_i|y_0, \ldots, y_i)$ as $\overline{X}_i^+$

the standard deviation of $P(X_i|y_0, \ldots, y_{i-1})$ as σ_i^-

of $P(X_i|y_0, \ldots, y_i)$ as σ_i^+

Dynamic Model:

$$x_i \sim N(d_i x_{i-1}, \sigma_{d_i})$$

$$y_i \sim N(m_i x_i, \sigma_{m_i})$$

Start Assumptions: $\overline{x}_0^-$ and σ_0^- are known
Update Equations: Prediction

$$\overline{x}_i^- = d_i \overline{x}_{i-1}^+$$

$$\sigma_i^- = \sqrt{\sigma_{d_i}^2 + (d_i \sigma_{i-1}^+)^2}$$

Update Equations: Correction

$$x_i^+ = \left(\frac{\overline{x}_i^- \sigma_{m_i}^2 + m_i y_i (\sigma_i^-)^2}{\sigma_{m_i}^2 + m_i^2 (\sigma_i^-)^2} \right)$$

$$\sigma_i^+ = \sqrt{\left(\frac{\sigma_{m_i}^2 (\sigma_i^-)^2}{(\sigma_{m_i}^2 + m_i^2 (\sigma_i^-)^2)} \right)}$$

Dynamic Model:

$$\boldsymbol{x}_i \sim N(\mathcal{D}_i \boldsymbol{x}_{i-1}, \Sigma_{d_i})$$

$$\boldsymbol{y}_i \sim N(\mathcal{M}_i \boldsymbol{x}_i, \Sigma_{m_i})$$

Start Assumptions: $\overline{\boldsymbol{x}}_0^-$ and Σ_0^- are known
Update Equations: Prediction

$$\overline{\boldsymbol{x}}_i^- = \mathcal{D}_i \overline{\boldsymbol{x}}_{i-1}^+$$

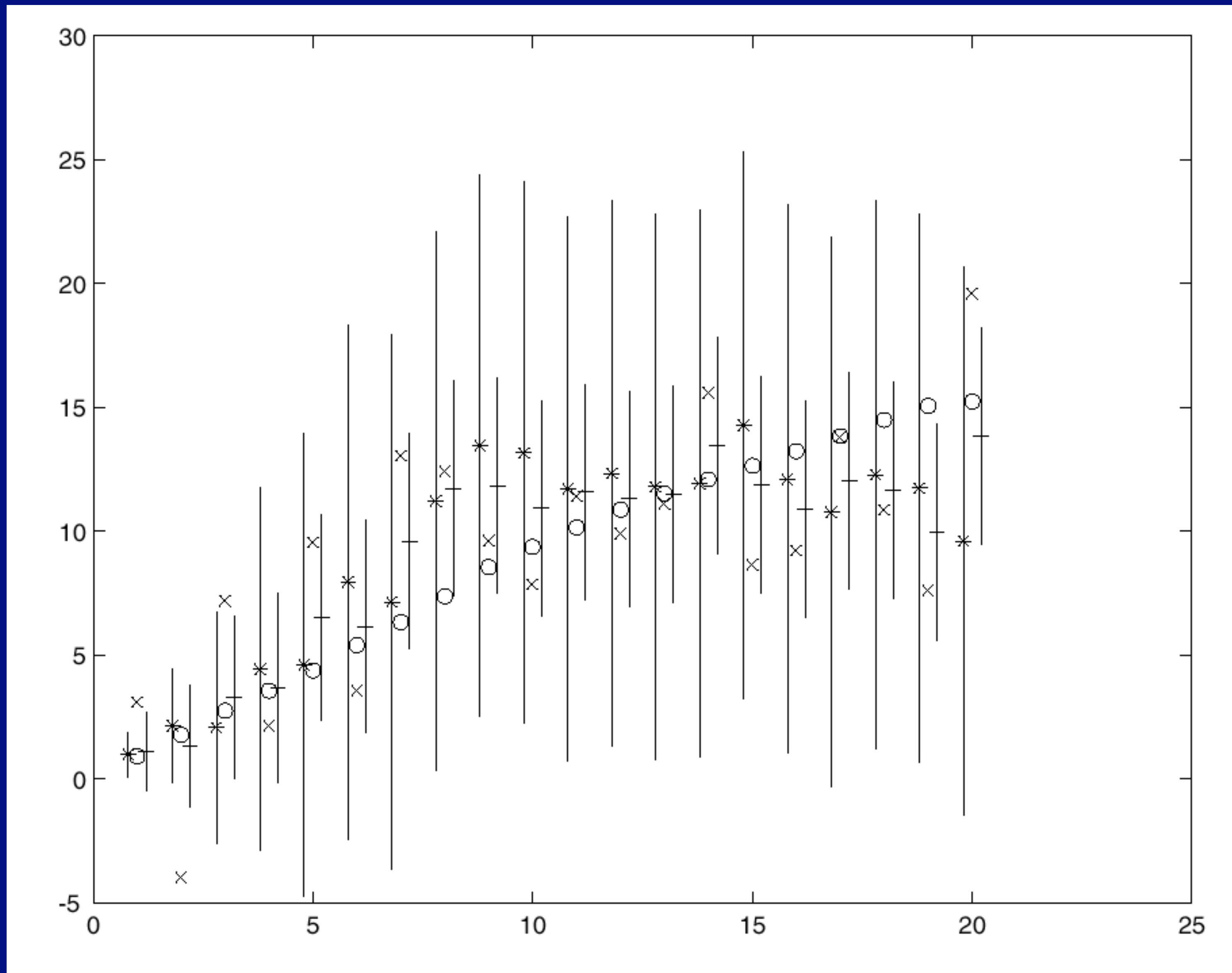
$$\Sigma_i^- = \Sigma_{d_i} + \mathcal{D}_i \sigma_{i-1}^+ \mathcal{D}_i$$

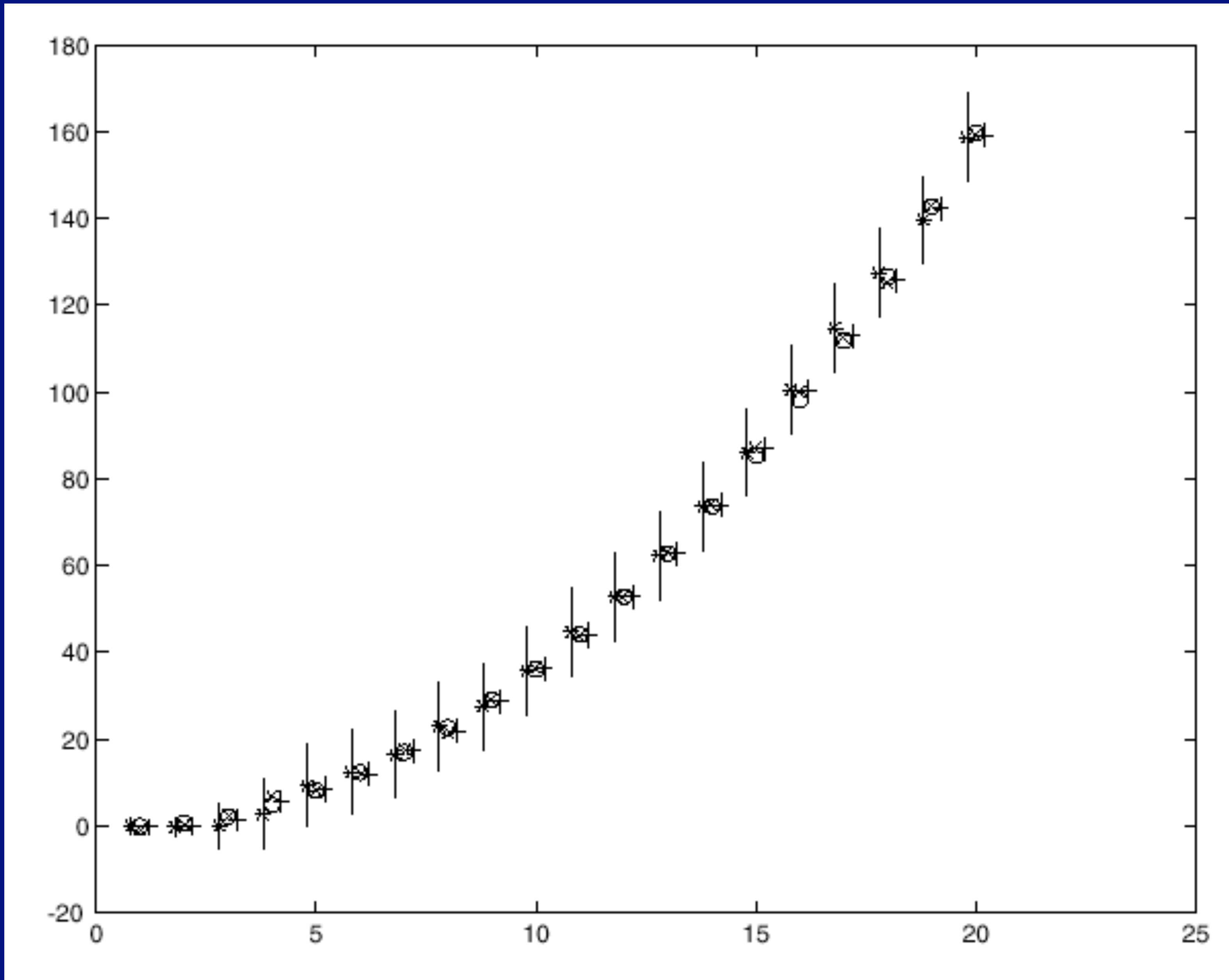
Update Equations: Correction

$$\mathcal{K}_i = \Sigma_i^- \mathcal{M}_i^T \left[\mathcal{M}_i \Sigma_i^- \mathcal{M}_i^T + \Sigma_{m_i} \right]^{-1}$$

$$\overline{\boldsymbol{x}}_i^+ = \overline{\boldsymbol{x}}_i^- + \mathcal{K}_i \left[\boldsymbol{y}_i - \mathcal{M}_i \overline{\boldsymbol{x}}_i^- \right]$$

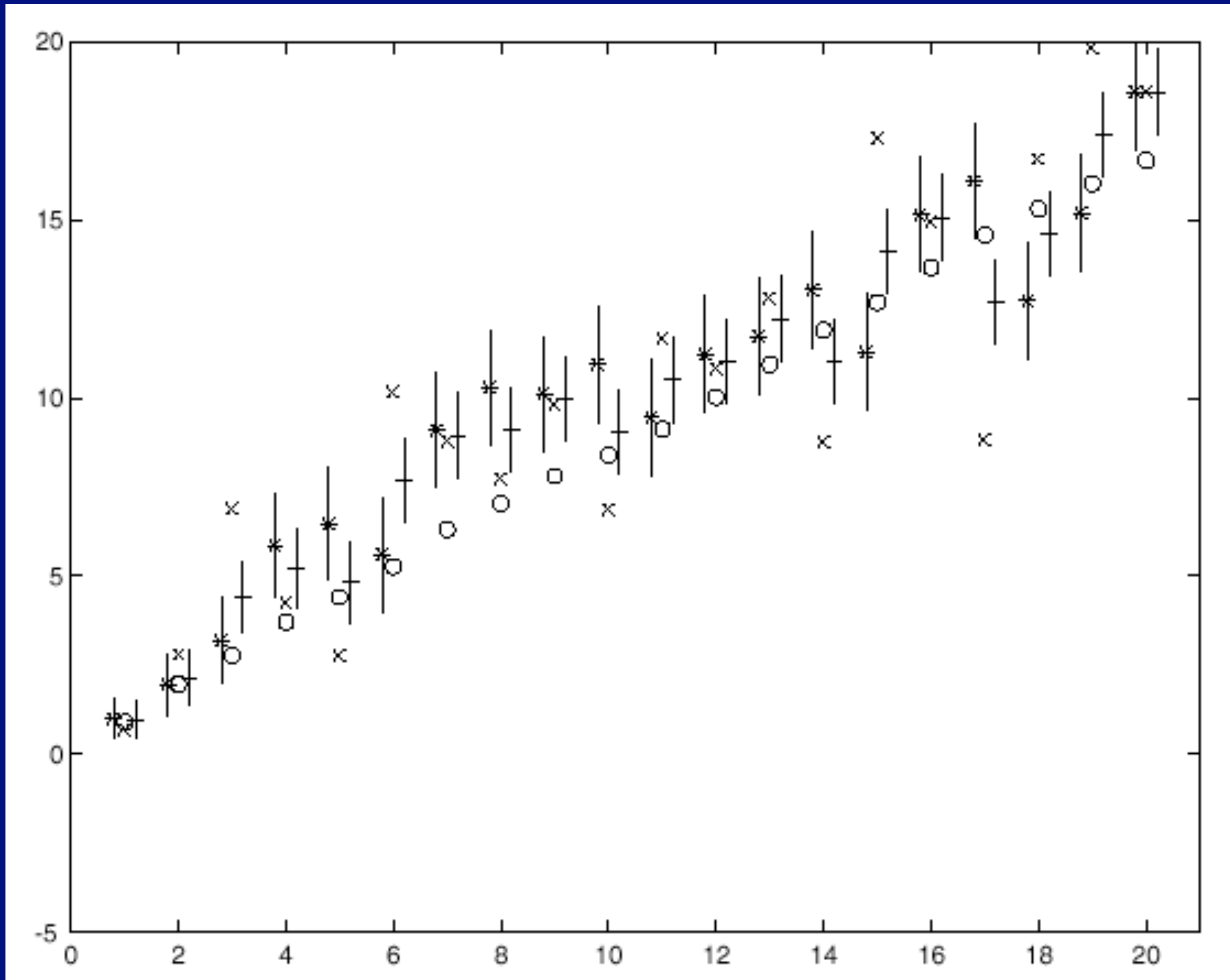
$$\Sigma_i^+ = \left[Id - \mathcal{K}_i \mathcal{M}_i \right] \Sigma_i^-$$


30
25
20
15
10
5
0
-5
0
5
10
15
20
25

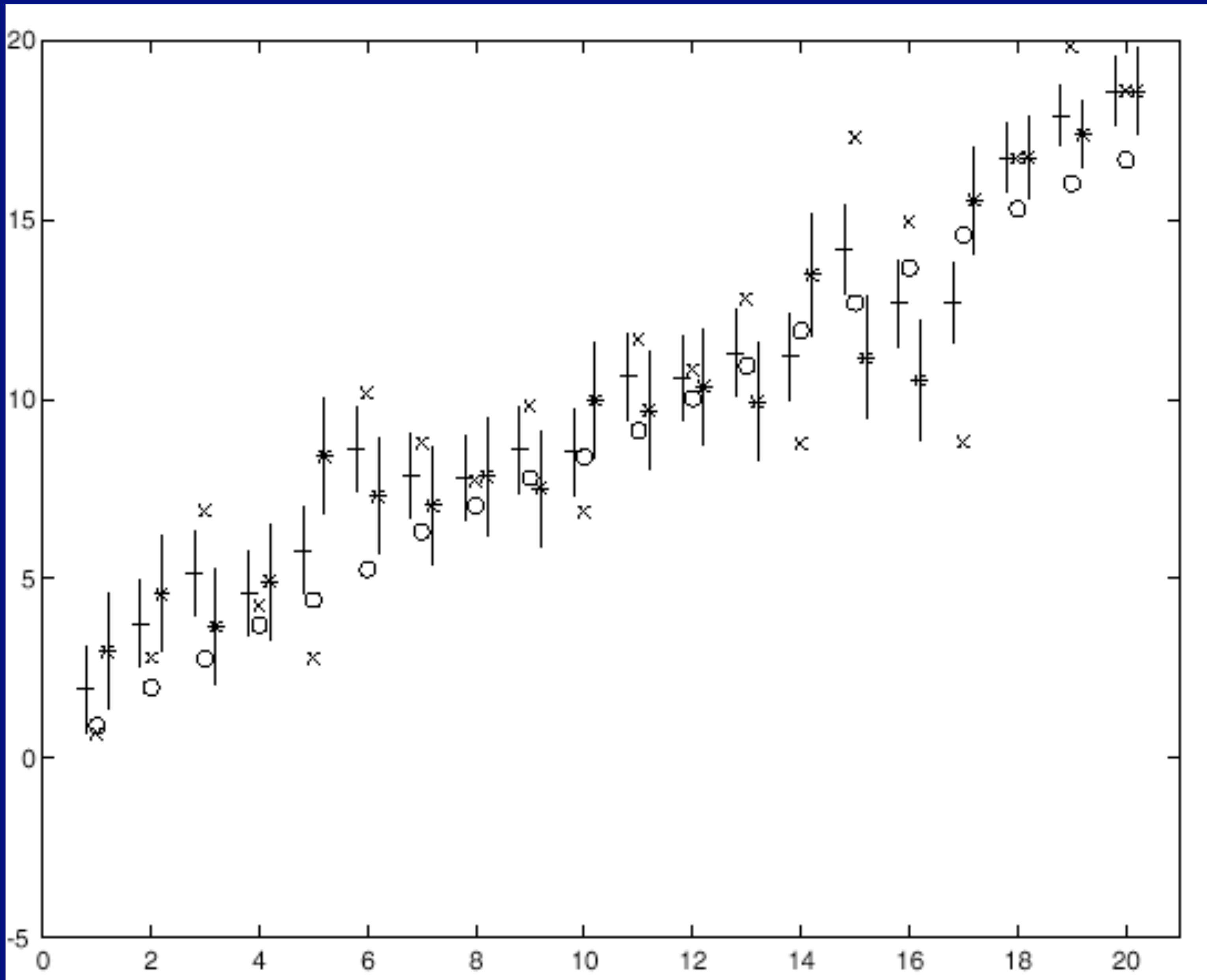

180
160
140
120
100
80
60
40
20
0
-20
0
5
10
15
20
25

Smoothing

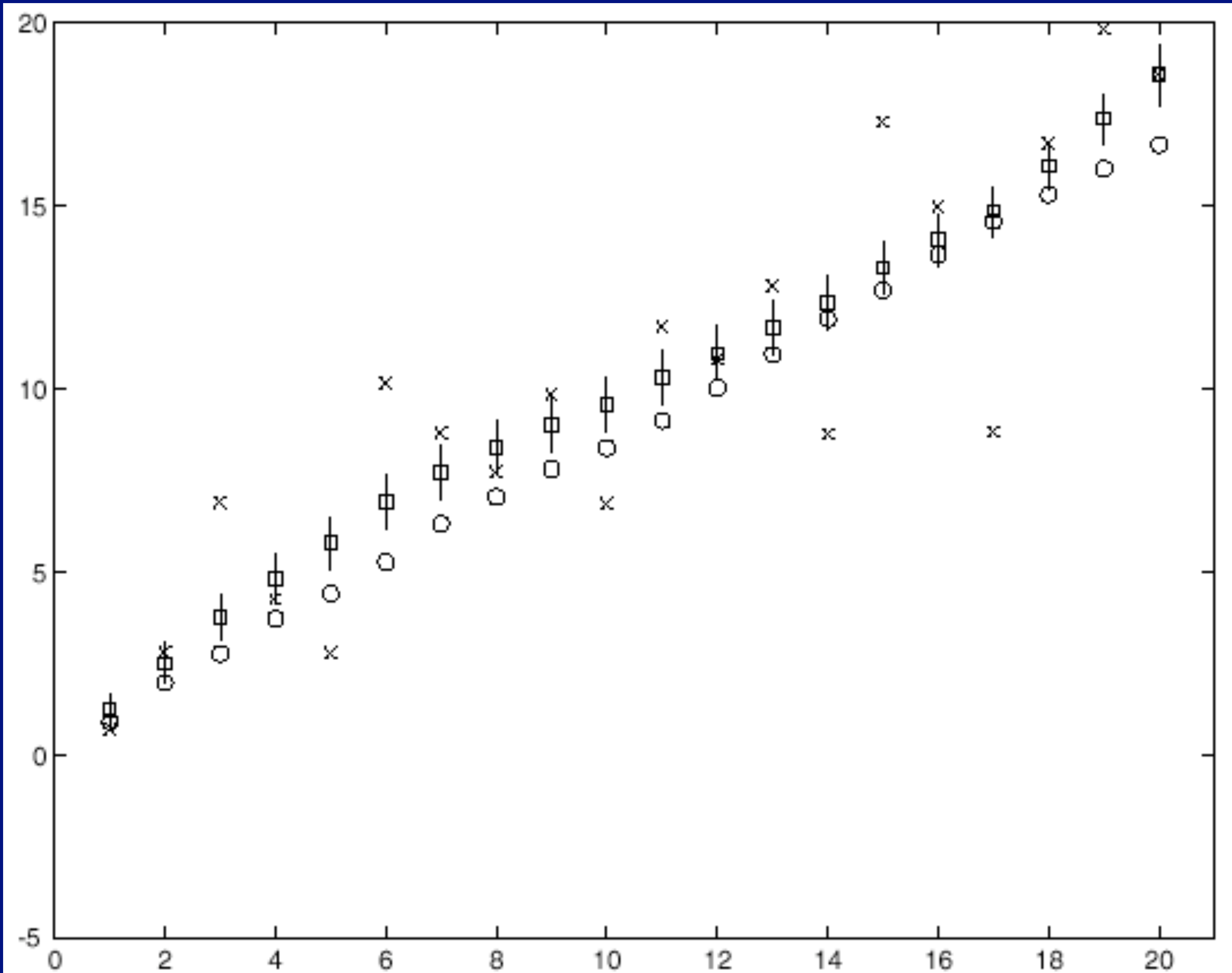
- Idea
  - We don’t have the best estimate of state - what about the future?
  - Run two filters, one moving forward, the other backward
  - Now combine state estimates



20
15
10
5
0
-5
0
2
4
6
8
10
12
14
16
18
20



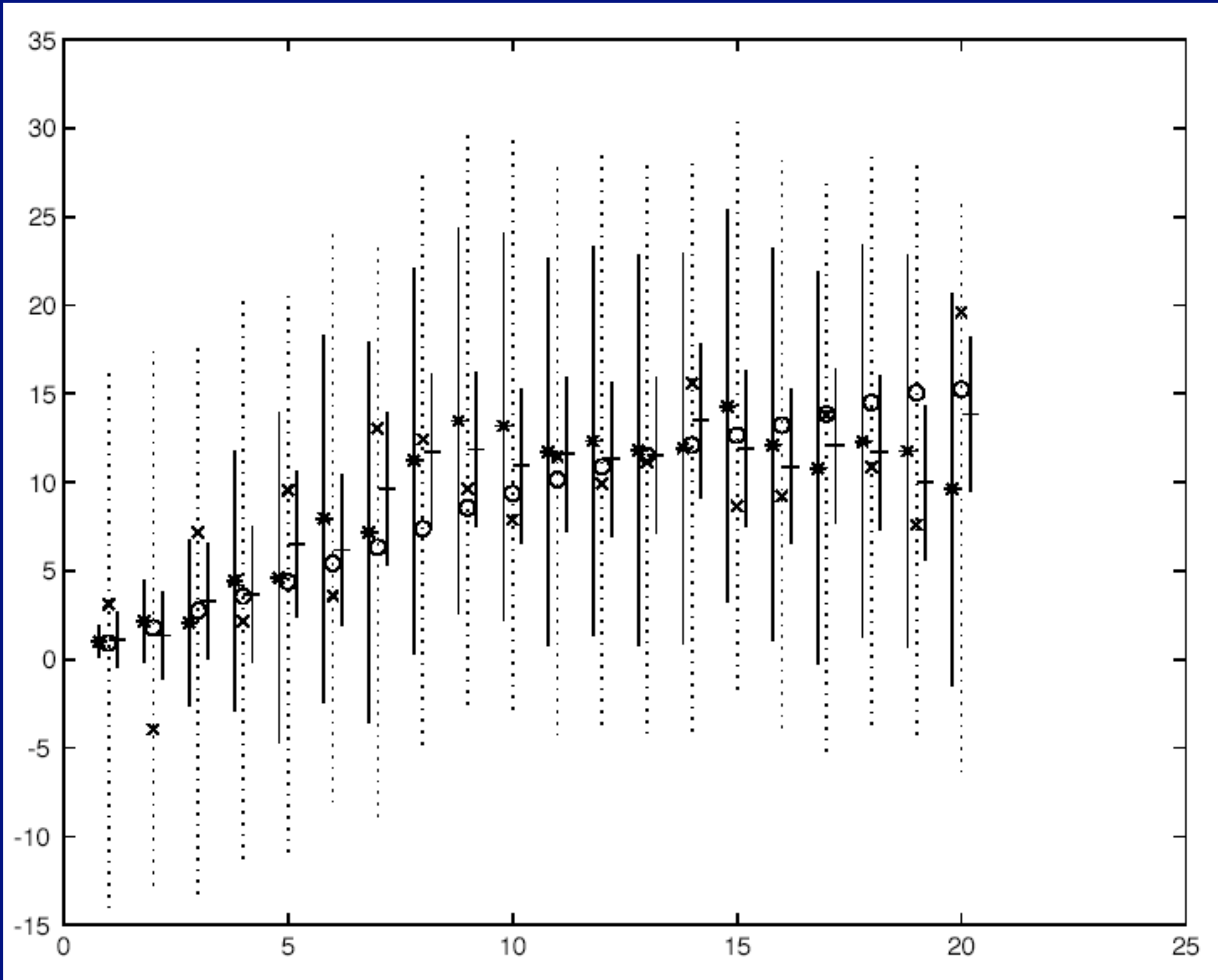
20
15
10
5
0
-5
0
2
4
6
8
10
12
14
16
18
20

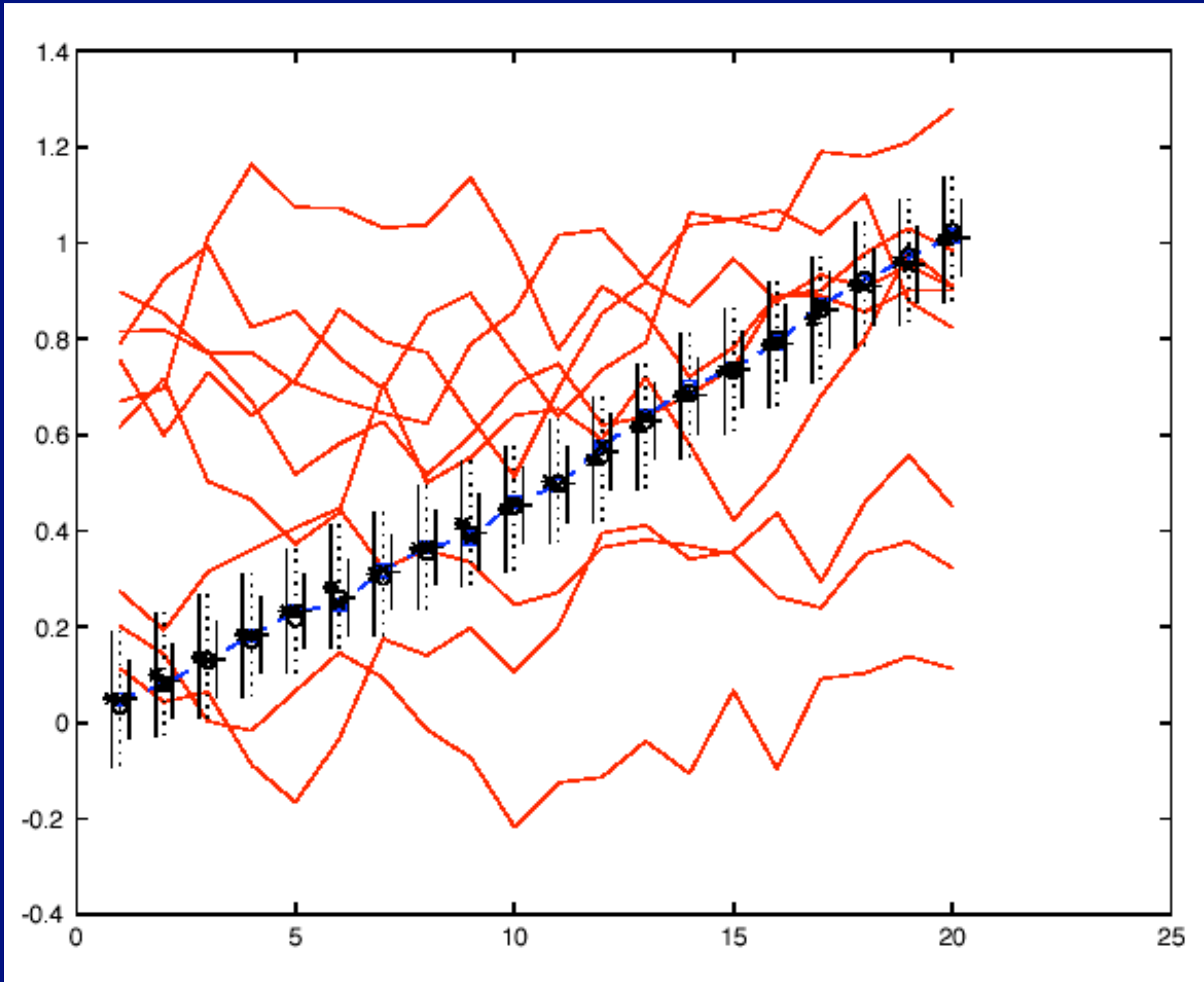


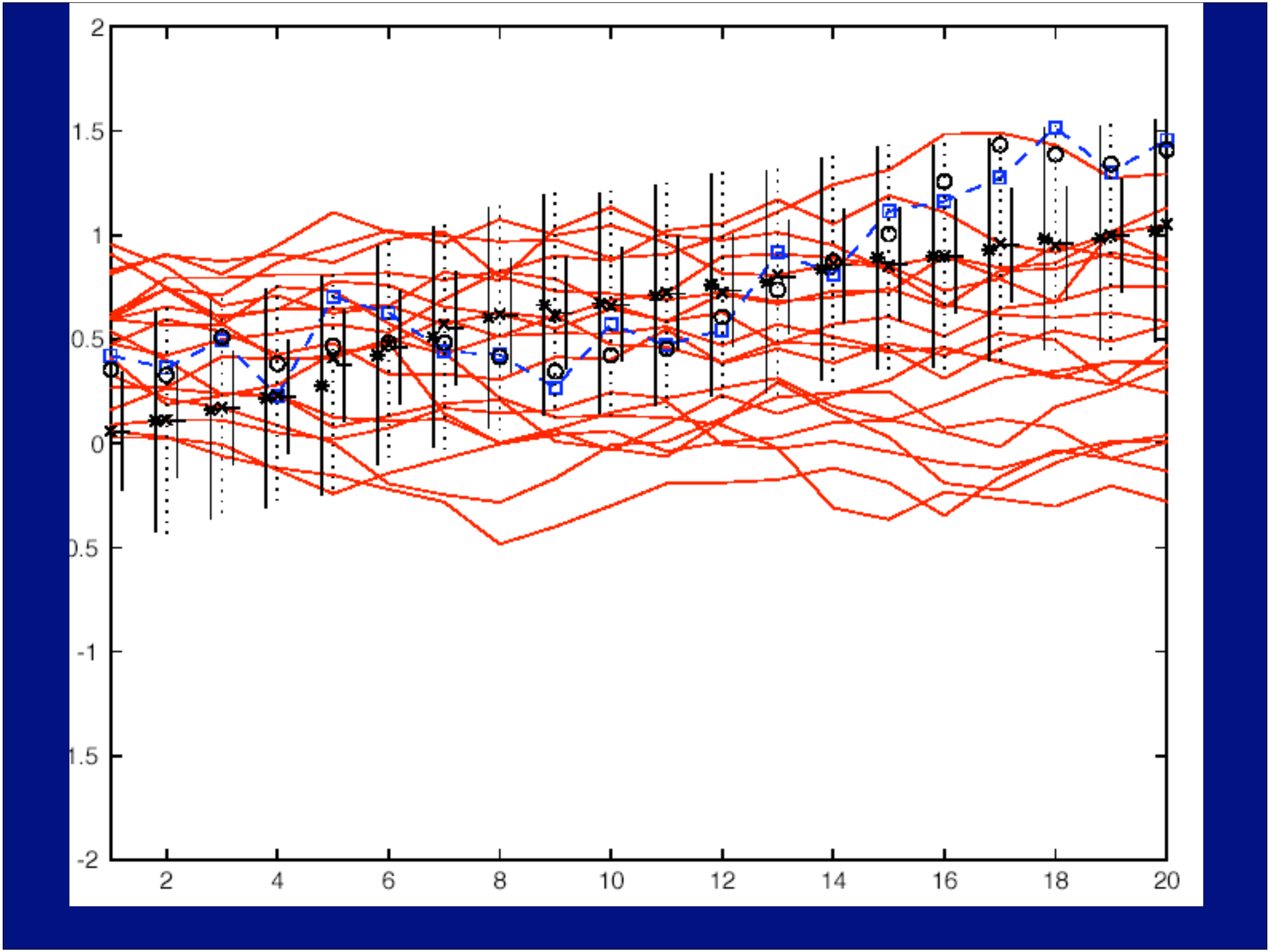
20
15
10
5
0
-5
0
2
4
6
8
10
12
14
16
18
20

Data Association

- Nearest neighbours
  - choose the measurement with highest probability given predicted state
  - popular, but can lead to catastrophe
- Probabilistic Data Association
  - combine measurements, weighting by probability given predicted state
  - gate using predicted state


35
30
25
20
15
10
5
0
-5
-10
-15
0
5
10
15
20
25


1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
5
10
15
20
25



2
1.5
1
0.5
0
0.5
-1
1.5
-2
2
4
6
8
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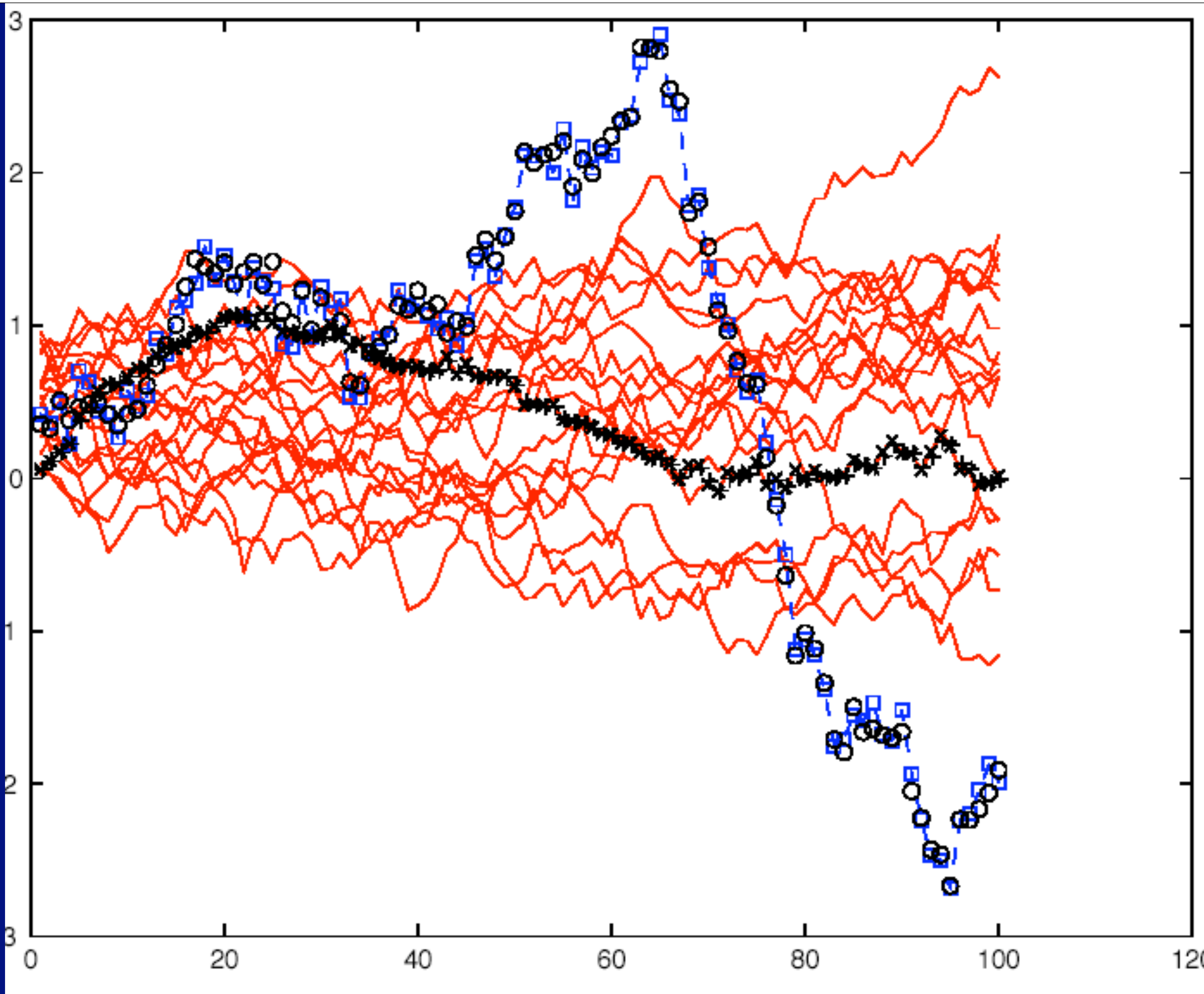
18
20



3
2
1
0
1
2
3
0
20
40
60
80
100
120

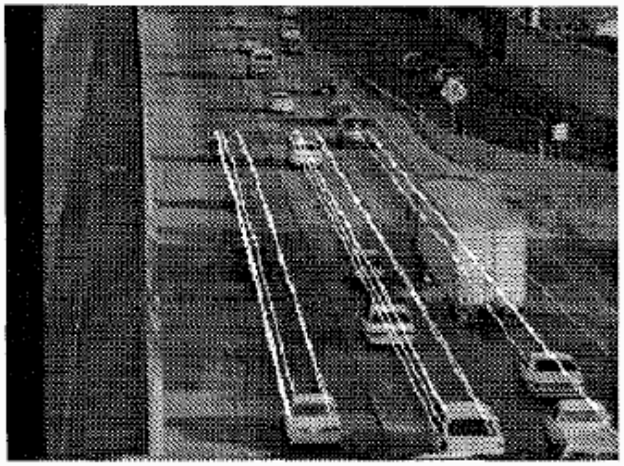


Figure 4: Example tracks of corner features.

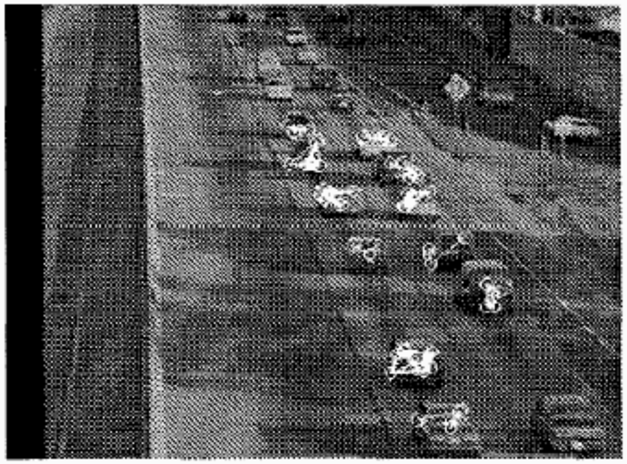
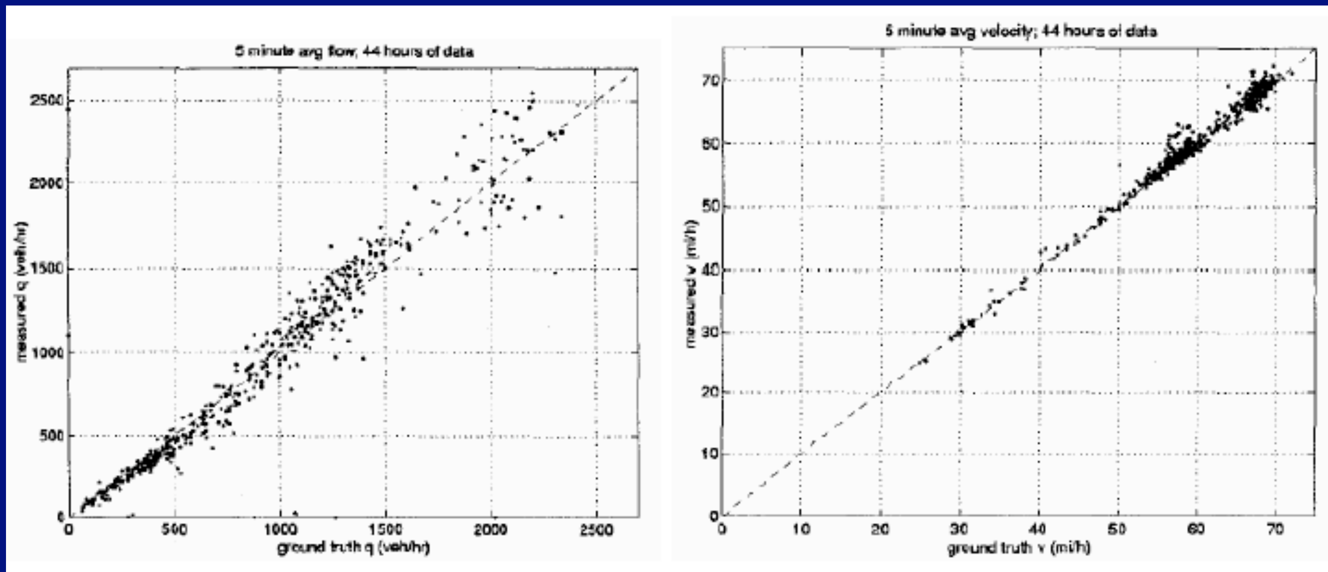


Figure 5: Example groups of corner features.

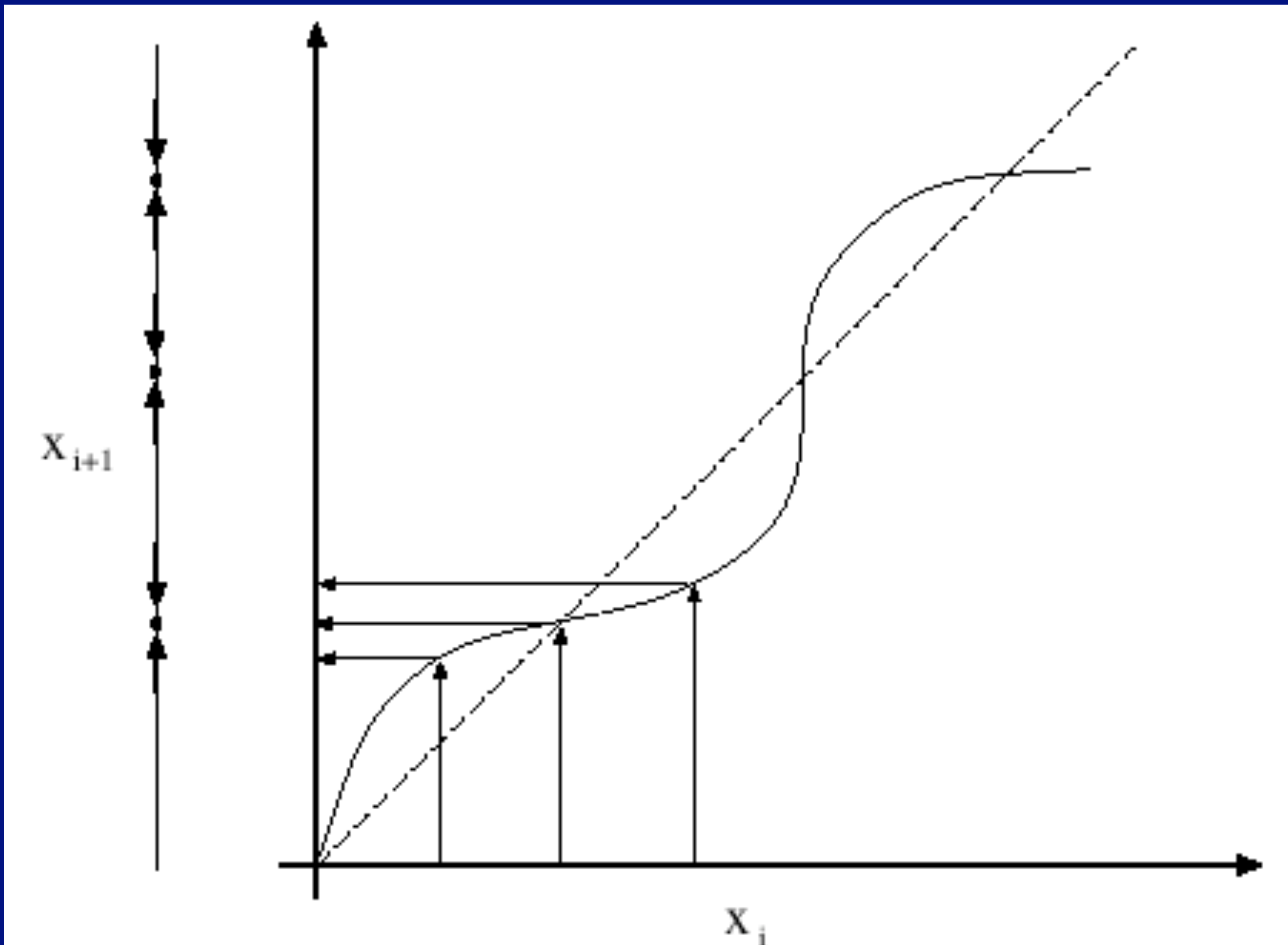
resent vehicles. *figure from A Real-Time Computer Vision System for Measuring Traffic Parameters, Beymer, McClachlan, Coifman and Malik et al. p.498, in the fervent hope of receiving permission*

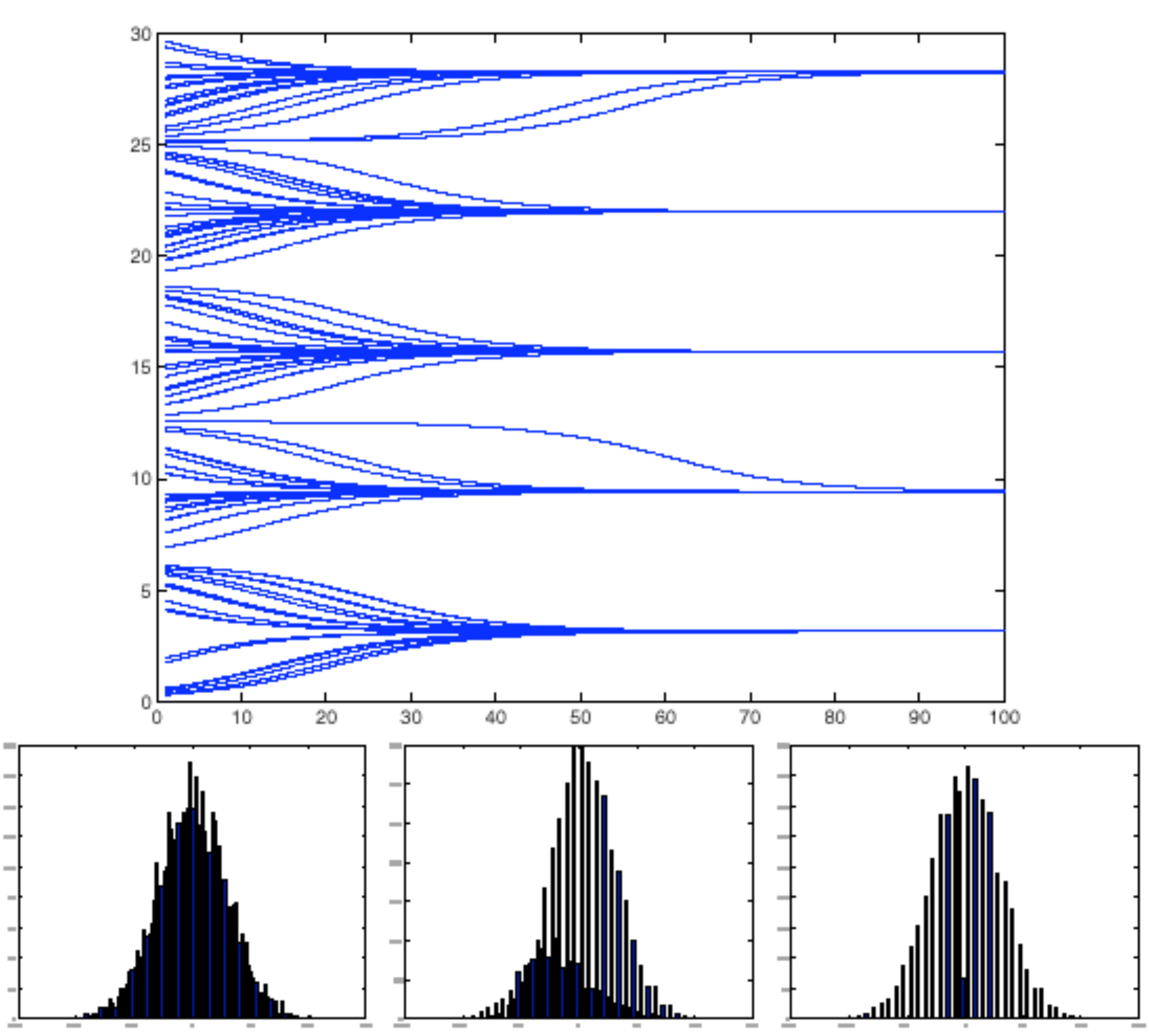


from A Real-Time Computer Vision System for Measuring Traffic Parameters, Beymer, McClachlan, Coifman and Malik et al. p.500, in the fervent hope of receiving permission

Beyond the Kalman Filter

- Various phenomena lead to multiple modes
  - nonlinear dynamics
  - kinematic ambiguities
  - data association problems
- Kalman filters represent these poorly
  - alternatives
    - Mixture models
    - particle filters


X_{i+1}
X_i


30
25
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0
10
20
30
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50
60
70
80
90
100

Multiple Modes from Data Association

- Linear dynamics, Linear measurement, two measurements
  - Both Gaussian, one depends on state and other doesn’t
  - Not known which depends on state
- One hidden variable per frame
- Leads to 2^(number of frames) mixture of Gaussians